

Capacitor Bank Unbalance Protection Calculations and Sensitivity Analysis

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Outline

- Common bank configurations and unbalance protection methods
- Capacitor unit arrangement, failure model, and the per-unit system
- Review of unbalance calculation equations
- Unbalance derivations explained
- Application to settings calculations
- Analysis and insights



Common bank configurations Grounded single-wye



- 87V
- 50Q / 51Q / 50QT
- 21C

Common bank configurations Ungrounded single-wye



- 59NU
- 59NT
- 50Q / 51Q / 50QT

Common bank configurations Grounded double-wye



- 87V (tap-bus, tap-tap)
- 60P
- 60N
- 50Q / 51Q / 50QT
- 21C

Common bank configurations Ungrounded double-wye



- 87V (tap-tap)
- 60P
- 60N
- 59NU
- 59NT
- 50Q / 51Q / 50QT

Common bank configurations Grounded H-bridge



- 87V (tap-bus)
- 60P
- 60N
- 50Q / 51Q / 50QT
- 21C

Common bank configurations Ungrounded H-bridge



Unbalance protection

- 60P
- 60N
- 59NU
- 59NT

60P

• 50Q / 51Q / 50QT

Motivation Present approach

- Setting unbalance protection elements (alarm, trip) requires calculating their operating signals for unit failures
- Short-circuit programs do not support capacitor bank failure calculations
- Step-by-step calculations or home-made software used instead
- Weak justification for selecting alarm vs. trip thresholds



Motivation Our approach

- One-step calculations for
 - All common bank configurations
 - All applicable unbalance protection elements
 - Fail-open and fail-short scenarios
 - Unbalance signal as (1) a function of failure size and (2) a function of healthy unit overvoltage
- Settings calculations
 - Alarm based on failure size
 - Trip before overvoltage breaches unit voltage rating



What is a one-step calculation? Ungrounded double-wye bank



Basic bank data:

- P number of units in a group
- S number of groups in a string
- R number of strings in a phase
- T per-unit 87V tap position

Basic failure data:

F number of failed units Fused bank (fail-open scenario)

What is a one-step calculation? Ungrounded double-wye bank



$$V_{59N} = \frac{F}{6SPR - F(6SR - 6R + 1)}$$

$$V_{87} = \frac{6TF}{6SRP - F\left(6SR - \frac{6R - 5T - 1}{1 - T}\right)}$$

$$3I_2 = \frac{3F}{6SPR - F(6SR - 6R + 1)}$$

$$I_{60P} = \frac{3F}{6SPR - F(6SR - 6R + 1)}$$

$$I_{60N} = \frac{1}{2}\frac{3F}{6SPR - F(6SR - 6R + 1)}$$

Capacitor unit arrangement



- For simplicity and symmetry, we use the same model for
 - Fuseless banks
 - Internally fused banks
 - Externally fused banks
- Not all combinations apply to practical banks, but
- All practical banks are covered by our calculations

Capacitor unit failure models Fail-open scenario



Capacitor unit failure models Fail-short scenario



Per-unit system

- Per-unit short-circuit calculations serve us well in many applications
- In per unit, the capacitor unbalance calculations simplify considerably
- In per unit, it is easy to spot errors and see patterns

Base voltage phase-to-ground system nominal voltage

Base current

bank current under nominal system voltage

Base reactance

base voltage / base current

Secondary value

per-unit value · base value / instrument transformer ratio

Key output of this work

- Appendix A
- One page for each bank configuration
- Fail-open and fail-short columns
- Self-contained reference material

Fail-Onen	Fail-Short
Tan-opti	Fair-Short
$k_{OV} = \frac{SP}{SP - F(S - 1)}$	$k_{OV} = \frac{S}{S - F}$
$F = \frac{SP}{S-1} \frac{k_{OV} - 1}{k_{OV}}$	$F = S \frac{k_{OV} - 1}{k_{OV}}$
$3I_2 = \frac{F}{SRP - F(SR - R)} 1\angle -90^{\circ}$	$3I_2 = \frac{F}{SR - FR} 1 \angle 90^{\circ}$
$3I_2 = \frac{k_{OV} - 1}{R(S - 1)} 1 \angle -90^{\circ}$	$3I_2 = \frac{k_{OV} - 1}{R} 1 \angle 90^{\circ}$
$\Delta X_{PHASE} = \frac{F}{SRP - F(SR - R + 1)}$	$\Delta X_{\text{PHASE}} = -\frac{F}{SR - F(R - 1)}$
$\Delta X_{\text{PHASE}} = \frac{k_{\text{OV}} - 1}{R(S - 1) + 1 - k_{\text{OV}}}$	$\Delta X_{\text{PHASE}} = -\frac{k_{\text{OV}} - 1}{R + k_{\text{OV}} - 1}$
$\Delta X_{\text{STRING}} = R \frac{F}{\text{SP} - \text{FS}}$	$\Delta X_{\text{STRING}} = -\frac{R}{S}F$
$\Delta X_{\text{STRING}} = R \frac{k_{\text{OV}} - 1}{S - k_{\text{OV}}}$	$\Delta X_{\text{STRING}} = -R \frac{k_{\text{OV}} - 1}{k_{\text{OV}}}$
$\Delta V_{g7} = \frac{TF}{SRP - F\left(SR - \frac{R - T}{1 - T}\right)} 1 \angle 0^{\circ}$	$\Delta V_{g7} = \frac{TF}{SR - F\frac{R - T}{1 - T}} 1 \angle 180^{\circ}$
$\Delta V_{87} = \frac{T(1-T)(k_{OV}-1)}{R(S-1) - T(SR - (R-1)k_{OV} - 1)} 1 \angle 0^{\circ}$	$\Delta V_{g7} = \frac{T(1-T)(k_{OV}-1)}{R-T((R-1)k_{OV}+1)} 1 \angle 180^{\circ}$
Notes:	
$\begin{pmatrix} 50 \\ Q \end{pmatrix} \begin{pmatrix} 21 \\ C \end{pmatrix}$ All values are in per unit.	
ΔX_{PHASE} and ΔX_{STRING} are both in per unit of the bank reactance.	
$\stackrel{\bullet}{\leftarrow}$ $\stackrel{\bullet}{\leftarrow}$ $\stackrel{\bullet}{\leftarrow}$ Voltage and current phase angles are relative to the faulted-phase voltage.	
$ T = \frac{X_{BOTTOM}}{X_{TOR} + X_{BOTTOM}} $	
ΔV_{87} \Box	
are parameters of the top pa	

Unbalance derivations V_{59N} in an ungrounded single-wye bank

V_A V_B V_C $V_{\text{A}}-V_{59\text{N}}$ В V_{59N}

 $I_{A} + I_{B} + I_{C} = 0$

$$\frac{V_{A} - V_{59N}}{-jX_{F}} + \frac{V_{B} - V_{59N}}{-jX} + \frac{V_{C} - V_{59N}}{-jX} = 0$$

$$V_{\rm B} = a^2 \cdot V_{\rm A}$$
, $V_{\rm C} = a \cdot V_{\rm A}$, $a = 1 \angle 120^\circ$

$$V_{59N} \cdot \left(\frac{1}{X_F} + \frac{2}{X}\right) = V_A \cdot \left(\frac{1}{X_F} + \frac{a^2 + a}{X}\right)$$

$$V_{59N} \cdot \left(\frac{X + 2 \cdot X_F}{X \cdot X_F}\right) = V_A \cdot \left(\frac{X - X_F}{X \cdot X_F}\right)$$

$$V_{59N} = V_A \cdot \left(\frac{X - X_F}{X + 2 \cdot X_F}\right)$$

Unbalance derivations V_{59N} in an ungrounded single-wye bank



$$V_{59N} = V_A \cdot \left(\frac{X - X_F}{X + 2 \cdot X_F}\right)$$

$$X = \frac{X_U}{P} \cdot S \cdot \frac{1}{R}$$

$$(X_F)^{-1} = \left(\frac{X_U}{P-F} + \frac{X_U}{P} \cdot (S-1)\right)^{-1} + \left(\frac{X_U}{P} \cdot S \cdot \frac{1}{R-1}\right)^{-1}$$

$$V_{59N(PU)} = \frac{F}{3 \cdot S \cdot P \cdot R - F \cdot (3 \cdot S \cdot R - 3 \cdot R + 1)} \cdot 1 \angle 180^{\circ}$$

Unbalance derivations Methodology



Unbalance derivations Validation

- Inspect for expected results
- Inspect for symmetry with other equations
- Run RTDS simulations for all bank configurations by using different bank parameters
- Compare with known examples (e.g., C37.99, past projects)



Key output of this work

- Appendix A
- One page for each bank configuration
- Fail-open and fail-short columns
- Self-contained reference material

UNGROUNDED DOUBLE-WYE CAPACITOR BANK		
Fail-Open	Fail-Short	
$k_{ov} = \frac{6SPR}{6SPR - F(6SR - 6R + 1)}$ $F = \frac{6SPR}{2000} \frac{k_{ov} - 1}{1000}$	$k_{ov} = \frac{6SR}{6SR - F(6R - 1)}$ $F = \frac{6SR}{100} \frac{k_{ov} - 1}{100}$	
6SR - 6R + 1 k _{ov}	6R - 1 k _{ov}	
$3I_2 = \frac{3F}{6SPR - F(6SR - 6R + 1)} 1 \angle -90^{\circ}$	$3I_2 = \frac{3F}{6SR - F(6R - 1)}1\angle 90^\circ$	
$3I_2 = 3\frac{k_{ov} - 1}{6R(S - 1) + 1}1\angle -90^{\circ}$	$3I_2 = 3\frac{k_{OV} - 1}{6R - 1}1\angle 90^\circ$	
$V_{\text{59N}} = \frac{F}{6\text{SPR} - F(6\text{SR} - 6\text{R} + 1)} 1 \angle 180^{\circ}$	$V_{59N} = \frac{F}{6SR - F(6R - 1)} 1 \angle 0^{\circ}$	
$V_{59N} = \frac{k_{OV} - 1}{6R(S - 1) + 1} 1 \angle 180^{\circ}$	$V_{59N} = \frac{k_{OV} - 1}{6R - 1} 1 \angle 0^{\circ}$	
$V_{87} = \frac{6TF}{6SRP - F\left(6SR - \frac{6R - 5T - 1}{1 - T}\right)} 1 \angle 0^{\circ}$	$V_{87} = \frac{6TF}{6SR - F\frac{6R - 5T - 1}{1 - T}} 1 \angle 180^{\circ}$	
$V_{g7} = \frac{6T(1-T)(k_{OV}-1)}{6R(S-1) - T(6SR - 6(R-1)k_{OV}-5) + 1} 1 \angle 0^{\circ}$	$V_{g7} = \frac{6T(1-T)(k_{OV}-1)}{6R - T(6(R-1)k_{OV}+5) - 1} 1 \angle 180^{\circ}$	
I _{60P} = 21	$I_{60N} = 3I_2$	
I _{60N} =	$I_{60N} = \frac{1}{2} 3I_2$	
$V_{59N} = -j\frac{1}{3}3I_2$		
	Notes: All values are in per unit. Voltage and current phase angles are relative to the faulted-phase voltage. $T = \frac{X_{BOTTOM}}{X_{TOP} + X_{BOTTOM}}$ For nonhomogeneous banks (different unit arrangement above and below the tap), P and R are parameters of the top part and S is an equivalent value, as follows: $S = \frac{S_{TOP}}{1 - T}$	

Introducing overvoltage factor

 k_{OV} = <u>Voltage across the most stressed healthy unit due to a failure</u>

Unit voltage during nominal system conditions

- In the paper, we have
 - Derived $k_{\rm OV}$ as a function of failure size for all bank configurations and the fail-open and fail-short failure modes
 - Expressed each unbalance signal as a function of k_{OV}
- You can calculate the highest permissible k_{ov} and use it to calculate trip thresholds

$$k_{OV} = 1.1 \cdot \frac{V_{U}}{\left(\frac{V_{NOM}}{\sqrt{3} \cdot S}\right)} = 1.1 \cdot \frac{S \cdot V_{U}}{V_{BASE}}$$

Setting trip thresholds Ungrounded single-wye bank, fail-open example

$$k_{OV} = \frac{3SPR}{3SPR - F(3SR - 3R + 1)}$$

$$F = \frac{3SPR}{3SR - 3R + 1} \cdot \frac{k_{OV} - 1}{k_{OV}}$$

$$V_{59N} = \frac{F}{3SPR - F(3SR - 3R + 1)}$$

$$V_{59N} = \frac{k_{OV} - 1}{3R(S - 1) + 1}$$



Key output of this work

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UNGROUNDED DOUBLE-WYE CAPACITOR BANK		
Fail-Open	Fail-Short	
$k_{ov} = \frac{6SPR}{6SPR - F(6SR - 6R + 1)}$ $F = \frac{6SPR}{2000} \frac{k_{ov} - 1}{1000}$	$k_{ov} = \frac{6SR}{6SR - F(6R - 1)}$ $F = \frac{6SR}{100} \frac{k_{ov} - 1}{100}$	
6SR - 6R + 1 k _{ov}	6R - 1 k _{ov}	
$3I_2 = \frac{3F}{6SPR - F(6SR - 6R + 1)} 1 \angle -90^{\circ}$	$3I_2 = \frac{3F}{6SR - F(6R - 1)}1\angle 90^\circ$	
$3I_2 = 3\frac{k_{ov} - 1}{6R(S - 1) + 1}1\angle -90^{\circ}$	$3I_2 = 3\frac{k_{OV} - 1}{6R - 1}1\angle 90^\circ$	
$V_{\text{59N}} = \frac{F}{6\text{SPR} - F(6\text{SR} - 6\text{R} + 1)} 1 \angle 180^{\circ}$	$V_{59N} = \frac{F}{6SR - F(6R - 1)} 1 \angle 0^{\circ}$	
$V_{59N} = \frac{k_{OV} - 1}{6R(S - 1) + 1} 1 \angle 180^{\circ}$	$V_{59N} = \frac{k_{OV} - 1}{6R - 1} 1 \angle 0^{\circ}$	
$V_{87} = \frac{6TF}{6SRP - F\left(6SR - \frac{6R - 5T - 1}{1 - T}\right)} 1 \angle 0^{\circ}$	$V_{87} = \frac{6TF}{6SR - F\frac{6R - 5T - 1}{1 - T}} 1 \angle 180^{\circ}$	
$V_{g7} = \frac{6T(1-T)(k_{OV}-1)}{6R(S-1) - T(6SR - 6(R-1)k_{OV}-5) + 1} 1 \angle 0^{\circ}$	$V_{g7} = \frac{6T(1-T)(k_{OV}-1)}{6R - T(6(R-1)k_{OV}+5) - 1} 1 \angle 180^{\circ}$	
I _{60P} = 21	$I_{60N} = 3I_2$	
I _{60N} =	$I_{60N} = \frac{1}{2} 3I_2$	
$V_{59N} = -j\frac{1}{3}3I_2$		
	Notes: All values are in per unit. Voltage and current phase angles are relative to the faulted-phase voltage. $T = \frac{X_{BOTTOM}}{X_{TOP} + X_{BOTTOM}}$ For nonhomogeneous banks (different unit arrangement above and below the tap), P and R are parameters of the top part and S is an equivalent value, as follows: $S = \frac{S_{TOP}}{1 - T}$	

Grounded double-wye bank example Basic data

PARAMETER	VALUE
Voltage (kV LL)	138
Bus Voltage PTR	1200:1
Bank Nominal Power (MVAr)	100
Breaker CTR	2000:5
Units in a Group, P	15
Groups in a String, S	6
Strings in a Phase, R	1
Unit Voltage Rating, kV	13.8
Unit Type	Externally fused
87V Tap	One bottom group
87V Tap PTR	120:1
60N CTR	20:5



Grounded double-wye bank example Supporting variables

PARAMETER	VALUE
Voltage (kV LL)	138
Bus Voltage PTR	1200:1
Bank Nominal Power (MVAr)	100
Breaker CTR	2000:5
Units in a Group, P	15
Groups in a String, S	6
Strings in a Phase, R	1
Unit Voltage Rating, kV	13.8
Unit Type	Externally fused
87V Tap	One bottom group
87V Tap PTR	120:1
60N CTR	20:5

Highest permissible overvoltage factor

$$k_{OV} = 1.1 \cdot \frac{13.8}{\left(\frac{138}{\sqrt{3} \cdot 6}\right)} = 1.143$$

Tap position

$$T = \frac{1}{6} = 0.1667$$

Base voltage, current, and impedance

$$V_{BASE} = \frac{138 \text{ kV}}{\sqrt{3}} = 79.674 \text{ kV}$$
$$I_{BASE} = \frac{100 \text{ MVAr}}{\sqrt{3} \cdot 138 \text{ kV}} = 418.37 \text{ A}$$
$$Z_{BASE} = \frac{79.674 \text{ kV}}{418.37 \text{ A}} = 190.44 \Omega$$

PARAMETER	VALUE
Voltage (kV LL)	138
Bus Voltage PTR	1200:1
Bank Nominal Power (MVAr)	100
Breaker CTR	2000:5
Units in a Group, P	15
Groups in a String, S	6
Strings in a Phase, R	1
Unit Voltage Rating, kV	13.8
Unit Type	Externally fused
87V Tap	One bottom group
87V Tap PTR	120:1
60N CTR	20:5

$$3I_2 = I_{60N} = \frac{1}{2} \frac{F}{SPR - F(SR - R)} = \cdots$$
$$\dots = \frac{1}{2} \cdot \frac{1}{6 \cdot 15 \cdot 1 - 1 \cdot (6 \cdot 1 - 1)} = \frac{1}{170} = \cdots$$

$$3I_2 = 0.005882 \text{ pu} \cdot 418.37 \text{ A} \cdot \frac{5}{2000} = 0.0062 \text{ A sec}$$

 $I_{60N} = 0.005882 \text{ pu} \cdot 418.37 \text{ A} \cdot \frac{5}{20} = 0.6152 \text{ A sec}$

PARAMETER	VALUE
Voltage (kV LL)	138
Bus Voltage PTR	1200:1
Bank Nominal Power (MVAr)	100
Breaker CTR	2000:5
Units in a Group, P	15
Groups in a String, S	6
Strings in a Phase, R	1
Unit Voltage Rating, kV	13.8
Unit Type	Externally fused
87V Tap	One bottom group
87V Tap PTR	120:1
60N CTR	20:5

$$V_{87} = \frac{TF}{SRP - F\left(SR - \frac{R - T}{1 - T}\right)} = \cdots$$

$$= \frac{0.1667 \cdot 1}{6 \cdot 1 \cdot 15 - 1 \cdot \left(6 \cdot 1 - \frac{1 - 0.1667}{1 - 0.1667}\right)} = \cdots$$

... = 0.0019608 pu

...

$$V_{87} = 0.0019608 \text{ pu} \cdot 79.674 \text{ kV} \cdot \frac{1}{120} = 1.302 \text{ V sec}$$

PARAMETER	VALUE
Voltage (kV LL)	138
Bus Voltage PTR	1200:1
Bank Nominal Power (MVAr)	100
Breaker CTR	2000:5
Units in a Group, P	15
Groups in a String, S	6
Strings in a Phase, R	1
Unit Voltage Rating, kV	13.8
Unit Type	Externally fused
87V Tap	One bottom group
87V Tap PTR	120:1
60N CTR	20:5



Grounded double-wye bank example Trip thresholds (breaching unit voltage rating)

PARAMETER	VALUE
Voltage (kV LL)	138
Bus Voltage PTR	1200:1
Bank Nominal Power (MVAr)	100
Breaker CTR	2000:5
Units in a Group, P	15
Groups in a String, S	6
Strings in a Phase, R	1
Unit Voltage Rating, kV	13.8
Unit Type	Externally fused
87V Tap	One bottom group
87V Tap PTR	120:1
60N CTR	20:5

$$BI_2 = I_{60N} = \frac{k_{OV} - 1}{2R(S - 1)} = \frac{1.143 - 1}{2 \cdot 1 \cdot (6 - 1)} = 0.0143 \text{ pu}$$

$$3I_2 = 0.0143 \text{ pu} \cdot 418.37 \text{ A} \cdot \frac{5}{2000} = 0.0150 \text{ A sec}$$

$$I_{60N} = 0.0143 \text{ pu} \cdot 418.37 \text{ A} \cdot \frac{5}{20} = 1.50 \text{ A sec}$$

Grounded double-wye bank example Trip thresholds (breaching unit voltage rating)

PARAMETER	VALUE
Voltage (kV LL)	138
Bus Voltage PTR	1200:1
Bank Nominal Power (MVAr)	100
Breaker CTR	2000:5
Units in a Group, P	15
Groups in a String, S	6
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Unit Voltage Rating, kV	13.8
Unit Type	Externally fused
87V Tap	One bottom group
87V Tap PTR	120:1
60N CTR	20:5



 Some unbalance protection operating signals have identical or proportional values

Examples

Any ungrounded bank

$$V_{59N} = -j\frac{1}{3} \cdot 3I_2$$

Double-wye grounded bank

$$I_{60P} = I_{60N} = 3I_2$$

Double-wye ungrounded bank

$$I_{60P} = 3I_2, \qquad I_{60N} = \frac{1}{2} \cdot 3I_2$$



- Some unbalance protection operating signals have identical or proportional values
- All unbalance protection operating signals are very similar in per-unit values

Example

Grounded single-wye bank, fail open:

$$|3I_2| = \frac{F}{S \cdot R \cdot P - F \cdot (S \cdot R - R)}$$
$$|\Delta X| = \frac{F}{S \cdot R \cdot P - F \cdot (S \cdot R - R + 1)}$$
$$|\Delta X| \simeq |3I_2|$$



- Some unbalance protection operating signals have identical or proportional values
- All unbalance protection operating signals are very similar in per-unit values
- All unbalance equations have a common format

Example

$$|V_{59N}| = \frac{F}{3 \cdot S \cdot P \cdot R - F \cdot (3 \cdot S \cdot R - 3 \cdot R + 1)}$$
$$|3I_2| = \frac{1}{2} \cdot \frac{F}{S \cdot R - F \cdot R}$$



F

- Some unbalance protection operating signals have identical or proportional values
- All unbalance protection operating signals are very similar in per-unit values
- All unbalance equations have a common format
- Unbalance protection operating signals can be approximated as follows

 $Y_{(PU)} \approx k \cdot \frac{\text{Number of Lost Units}}{\text{Number of Units per Phase}}$



PARAMETER	VALUE
Voltage (kV LL)	138
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Unit Voltage Rating, kV	13.8
Unit Type	Externally fused
87V Tap	One bottom group
87V Tap PTR	120:1
60N CTR	20:5

$$3I_2 = I_{60N} = \frac{1}{2} \frac{F}{SPR - F(SR - R)} = \cdots$$
$$\dots = \frac{1}{2} \cdot \frac{1}{6 \cdot 15 \cdot 1 - 1 \cdot (6 \cdot 1 - 1)} = \frac{1}{170} = 0.0059 \text{ pu}$$

$$Y_{(PU)} \approx \frac{1}{6 \cdot 15 \cdot 2} = \frac{1}{180} = 0.0056 \text{ pu}$$

The approximation is accurate within 6 percent

Additional material in the paper

- Calculations for multiple capacitor unit failures
- Calculations for partial capacitor unit failures (capacitor element failures)
- Optimizing for more sensitive unbalance protection
 - Unit arrangement
 - Tap and bridge position
- Discussion on self-canceling failures and redundancy of unbalance protection



Conclusions Calculations and analysis

- Capacitor unbalance calculations can be done in one step (single-equation calculations)
 - Simpler and faster
 - Less prone to errors
- Calculating in per unit allows greater insights and helps spot errors
- Unbalance signals can be easily approximated based on the count of failed units
- Calculations for multiple failures can be done by using the superposition principle



Conclusions Protection

- The paper teaches simple one-step calculations for both alarm and trip thresholds
- Setting trip thresholds based on the overvoltage factor is an elegant solution
- In per unit, all unbalance protection elements have near-identical sensitivities
 - Ease of measurement favors one element vs. others
 - Using multiple elements protects against multiple self-canceling failures

