Symmetrical Components and Fault Currents

\[ \mathbf{v}_{abc} = \begin{bmatrix} V_0 \\ V_0 \\ V_0 \end{bmatrix} + \begin{bmatrix} V_1 \\ \alpha^2 V_1 \\ \alpha V_1 \end{bmatrix} + \begin{bmatrix} V_2 \\ \alpha V_2 \\ \alpha^2 V_2 \end{bmatrix} \]

\[ = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \]

\[ = \mathbf{A} \mathbf{v}_{012} \]
Review of Phasors
Origin of Phasors
> Rotating rotors = alternating currents & voltages

> Phasors are well established means of representing ac circuits

Charles Proteus Steinmetz (1865-1923)
Phasors

Sine wave

Phasor representation of sine wave
Addition of Phasors
Multiplication of phasors

Multiplying two phasors results in another phasor. The magnitude of the new phasor is the product of the magnitude of the two original phasors. The angle of the new phasor is the sum of the angles of the two original phasors.
Review Of Symmetrical Components
Symmetrical and Non-Symmetrical Systems:

**Symmetrical System:**
- Counter-clockwise rotation
- All current vectors have equal amplitude
- All voltage phase vectors have equal amplitude
- All current and voltage vectors have 120 degrees phase shifts and a sum of 0V.

**Non-Symmetrical System:**
- Fault or Unbalanced condition
- If one or more of the symmetrical system conditions is not met
Symmetrical Components:

Positive Sequence (Always Present)

• A-B-C Counter-clockwise phase rotation
• All phasors with equal magnitude
• All phasors displaced 120 degrees apart

Zero Sequence

• No Rotation Sequence
• All phasors with equal magnitude
• All phasors are in phase

Negative Sequence

• A-C-B counter-clockwise phase rotation
• All phasors with equal magnitude
• All phasors displaced 120 degrees apart
Symmetrical Components:

**Positive Sequence Component:**
\[ I_1 = \frac{1}{3} (I_a + \alpha I_b + \alpha^2 I_c) \]
\[ V_1 = \frac{1}{3} (V_a + \alpha V_b + \alpha^2 V_c) \]

**Negative Sequence Component:**
\[ I_2 = \frac{1}{3} (I_a + \alpha^2 I_b + \alpha I_c) \]
\[ V_2 = \frac{1}{3} (V_a + \alpha^2 V_b + \alpha V_c) \]

**Zero Sequence Component:**
\[ I_0 = \frac{1}{3} (I_a + I_b + I_c) \]
\[ V_0 = \frac{1}{3} (V_a + V_b + V_c) \]

Unbalanced Line-to-Neutral Phasors:

\[ I_a = I_1 + I_2 + I_0 \]
\[ V_a = V_1 + V_2 + V_0 \]

\[ I_b = \alpha^2 I_1 + \alpha I_2 + I_0 \]
\[ V_b = \alpha^2 V_1 + \alpha V_2 + V_0 \]

\[ I_c = \alpha I_1 + \alpha^2 I_2 + I_0 \]
\[ V_c = \alpha V_1 + \alpha^2 V_2 + \alpha V_0 \]
Calculating Symmetrical Components:

Three-Phase Balanced / Symmetrical System

- Positive: \( V_a, \alpha^* V_b, \alpha^2 V_c \)
- Negative: \( \alpha^* V_c, 3V_2 = 0 \)
- Zero: \( 3V_0 = 0 \)

Open-Phase Unbalanced / Non-Symmetrical System

- Positive: \( I_a, \alpha^* I_b, \alpha^2 I_c \)
- Negative: \( \alpha^2 I_b, \alpha^* I_c \)
- Zero: \( 3I_0 \)
Symmetrical Components

Example: Perfectly Balanced & ABC Rotation

\[ I_0 = \frac{1}{3} (I_a + I_b + I_c) \]
\[ V_0 = \frac{1}{3} (V_a + V_b + V_c) \]
\[ I_1 = \frac{1}{3} (I_a + aI_b + a^2I_c) \]
\[ V_1 = \frac{1}{3} (V_a + aV_b + a^2V_c) \]
\[ I_2 = \frac{1}{3} (I_a + a^2I_b + aI_c) \]
\[ V_2 = \frac{1}{3} (V_a + a^2V_b + aV_c) \]

\[ a = 1 \leq 120^\circ \]
\[ a^2 = 1 \leq 240^\circ \]

Result: 100% I1 (Positive Sequence Component)
Symmetrical Components
Example: B-Phase Rolled & ABC Rotation

\[ I_0 = \frac{1}{3}(I_a + I_b + I_c) \quad V_0 = \frac{1}{3}(V_a + V_b + V_c) \]

\[ I_1 = \frac{1}{3}(I_a + aI_b + a^2I_c) \quad V_1 = \frac{1}{3}(V_a + aV_b + a^2V_c) \]

\[ I_2 = \frac{1}{3}(I_a + a^2I_b + aI_c) \quad V_2 = \frac{1}{3}(V_a + a^2V_b + aV_c) \]

\( a = 1 \leq 120^\circ \)

\( a^2 = 1 \leq 240^\circ \)

Result: 33% I1, 66% I0 and 66% I2
Symmetrical Components
Example: B-Phase & C-Phase Rolled & ABC Rotation

\[ l_0 = \frac{1}{3}l_a + l_b + l_c \]  
\[ v_0 = \frac{1}{3}v_a + v_b + v_c \]

\[ l_1 = \frac{1}{3}l_a + a l_b + a^2 l_c \]  
\[ v_1 = \frac{1}{3}v_a + a v_b + a^2 v_c \]

\[ l_2 = \frac{1}{3}l_a + a^2 l_b + a l_c \]  
\[ v_2 = \frac{1}{3}v_a + a^2 v_b + a v_c \]

\( a = 1 \leq 120^\circ \)  
\( a^2 = 1 \leq 240^\circ \)

Result: 100% I2 (Negative Sequence Component)
Power System Faults

Fault Analysis - Example:

For Fault Condition:
Positive Sequence Component, $I_1$:

$$I_1 = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c)$$

**Example:**

$I_a = 900 \text{ A}$
$\alpha I_b = 300 \text{ A}$
$\alpha^2 I_c = 300 \text{ A}$

Here, $\alpha = 120^\circ$ and $\alpha^2 = 240^\circ$.
Fault Analysis - Example

For Fault Condition:
Positive Sequence Component, $I_1$:

$$I_1 = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c)$$

$$= \frac{1}{3}(900 \text{ A} + 300 \text{ A} + 300 \text{ A})$$

$$= \frac{1}{3}(1500 \text{ A})$$

$I_1 = 500$ Amps
Power System Faults

Fault Analysis – Example:
For Fault Condition:
Negative Sequence Component, $I_2$:

\[ I_2 = \frac{1}{3}(I_a + \alpha^2 I_b + \alpha I_c) \]

\begin{align*}
I_a & = 900 \text{ A} \\
\alpha^2 I_b & = 300 \text{ A} \\
\alpha I_c & = 300 \text{ A}
\end{align*}

\[ \alpha^2 = 240^\circ \]

\[ \alpha = 120^\circ \]
Power System Faults

Fault Analysis - Example:
For Fault Condition:
Negative Sequence Component, I₂:

\[ I_2 = \frac{1}{3} (I_a + \alpha^2 I_b + \alpha I_c) \]

\[ = \frac{1}{3} (600 \text{ A}) \]

\[ I_2 = 200 \text{ Amps} \]

\[ \alpha^2 I_b = 300 \text{ A} \]

\[ \alpha I_c = 300 \text{ A} \]

\[ I_a = 900 \text{ A} \]
Power System Faults

Fault Analysis - Example:
For Fault Condition:
Zero Sequence Component, $I_0$:

$$I_0 = \frac{1}{3}(I_a + I_b + I_c)$$

900 A

$I_b$

300 A

$I_c$

300 A
Fault Analysis – Example:
For Fault Condition:
Zero Sequence Component, $I_0$:

$$I_0 = \frac{1}{3}(I_a + I_b + I_c)$$

$$= \frac{1}{3}(600 \text{ A})$$

$I_0 = 200$ Amps
Fault Currents
Power System
Sequence Networks

• Where is sequence voltage highest?
• What generates negative and zero sequence currents?
How do we connect the sequence networks for a particular fault?

• Assuming a Phase to Ground fault on a solidly grounded system, what do we know about the fault current?
  • Faulted phase is very large
  • Un-faulted phases are small, almost zero.
How do we connect the sequence networks for a particular fault?

- $I_a = x$
- $I_c = I_b = 0$
- $I_a + I_b + I_c = x = I_0$
- $I_a + aI_b + a^2I_c = x = I_1$
- $I_a + a^2I_b + aI_c = x = I_2$
- $I_1 = I_2 = I_0$
How do we connect so that $I_1 = I_2 = I_0$?

- The sequence networks have to be in series for a phase to ground fault on a solidly grounded system.
Phase to phase fault

• Assuming a Phase to Phase fault, what do we know about the fault current?
• Faulted phases are very large
• Un-faulted phase is small, almost zero.
How do we connect the sequence networks for a particular fault?

- \( I_c = 0 \)
- \( I_a = -I_b = x \)
- \( I_a + I_b + I_c = I_0 = 0 \)
- \( I_a + a^2I_b + aI_c = \sqrt{3}x = I_1 \)
- \( I_a + a^2I_b + aI_c = \sqrt{3}x = I_2 \)
- \( I_1 = I_2 \) and \( I_0 \) is not involved
How do we connect so that $I_1=I_2$ and $I_0=0$?

- The positive and negative sequence networks have to be in series for a phase to phase fault.
Three Phase Fault

• Assuming a three phase, what do we know about the fault current?
  • All three phase are large and equal
How do we connect the sequence networks for a particular fault?

- \( I_a = I_b = I_c = x \)
- \( \frac{1}{3}(I_a+I_b+I_c)=0=I_0 \)
- \( \frac{1}{3}(I_a+aI_b+a^2I_c)=x=I_1 \)
- \( \frac{1}{3}(I_a+a^2I_b+aI_c)=0=I_2 \)
- \( I_0=I_2 \) are not involved
How do we connect so that $I_2=I_0=0$

- The negative and zero sequence networks are not involved.
Line-to-line-to-ground fault

Assuming a B-phase to C-phase to ground, the conditions at the point of the fault are:

\[ I_A = 0 \]
\[ V_B = 0 \]
\[ V_C = 0 \]

Therefore, \( V_{o1} = V_{o2} = V_{o0} = \frac{V_A}{\sqrt{3}} \). Substituting into the voltage equations and simplifying, results in:

\[ I_{o1} = \frac{E_{o1}(Z_2 + Z_0)}{Z_1Z_2 + Z_1Z_0 + Z_2Z_0} \]

Therefore, all three sequence networks are connected. Since \( V_{o1} = V_{o2} = V_{o0} \), the networks are connected in parallel.
Common Fault Types:

- Balanced load or three-line-to-ground fault with impedances.
- A three-line-to-ground fault.
- A three-phase fault.
- A short circuit open.
- A line-to-ground fault through an impedance.
- A line-to-ground fault.
- A line-to-line fault through impedance.
- A line-to-line fault.
- A two-line-to-ground fault with impedance.
- A two-line-to-ground fault.
- A three-line-to-ground fault with impedance in phase a.
- Unbalanced load or three-line-to-ground fault with impedance.
Transformer Interconnections:

Two Winding Transformers

<table>
<thead>
<tr>
<th>Three Phase Connection</th>
<th>Zero Sequence Circuit</th>
<th>Positive or Negative Sequence</th>
</tr>
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<tbody>
<tr>
<td>e</td>
<td>![Diagram e]</td>
<td>![Diagram e]</td>
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<tr>
<td>b</td>
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<tr>
<td>h</td>
<td>![Diagram h]</td>
<td>![Diagram h]</td>
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</tbody>
</table>
Transformer Interconnections:

230-13KV
50MVA
Delta-Wye
5%

Zsystem

Zline
Networks with Transformers
What are my sequence impedances

- In transmission and distribution lines: \( Z_1 = Z_2 \). \( Z_0 \) is always different from \( Z_1 \) because it is a loop impedance (conductor + earth and or ground.) For transmission lines, \( X_1 \) is typically 2 to 6 times \( X_0 \).

- In transformers: \( Z_0 = Z_1 = Z_2 \) or \( Z_0 = \text{infinity} \) depending on the connection.
Calculate Phase to Ground Fault current on secondary of transformer, assuming an infinite bus
First, build the sequence network

- $Z_{t1} = 5\%$ on $50\text{MVA base}$
- $Z_{t2} = 5\%$ on $50\text{MVA base}$
- $Z_{t3} = 5\%$ on $50\text{MVA base}$
- $I = \frac{V}{Z} = 1\text{pu}/(0.05\text{pu} + 0.05\text{pu} + 0.05\text{pu})$
- $I = 6.667\text{pu}$
- $I_{\text{base}} = \frac{50\text{MVA}}{\sqrt{3} \times 13\text{KV}} = 2221\text{A}$
- $I_{1} = I_{2} = I_{0} = I_{\text{pu}} \times I_{\text{base}} = 14.8\text{KA}$
- $I_{a} = I_{1} + I_{2} + I_{0} = 44\text{KA}$
What would be the fault current with two transformers in parallel?

- $Z_{t1} = 2.5\%$ on 50MVA base
- $Z_{t2} = 2.5\%$ on 50MVA base
- $Z_{t3} = 2.5\%$ on 50MVA base
- $I = \frac{V}{Z} = \frac{1\text{pu}}{0.025\text{pu} + 0.025\text{pu} + 0.025\text{pu}}$
- $I = 13.3\text{pu}$
- $I_{\text{base}} = 50\text{MVA} / (\sqrt{3} \times 13\text{KV}) = 2221\text{A}$
- $I_1 = I_2 + I_3 = I_{\text{pu}} \times I_{\text{base}} = 30\text{KA}$
- $I_a = 90\text{KA}$
Calculate Three Phase Fault current on secondary of transformer, assuming an infinite bus
First, build the sequence network

- \( Z_{t1} = 5\% \) on 50MVA base
- \( I = \frac{V}{Z} = 1 \text{pu}/0.05\text{pu} \)
- \( I = 20\text{pu} \)
- \( I_{\text{base}} = \frac{50\text{MVA}}{\sqrt{3} \times 13\text{KV}} = 2221\text{A} \)
- \( I_1 = I_{\text{pu}} \times I_{\text{base}} = 44\text{KA} \)
- \( I_a = I_1 + I_2 + I_0 = 44\text{KA} \)

Phase to ground current is larger than phase to phase current at the transformer secondary terminals.
Calculate Phase to Ground Fault current on secondary of transformer, given a utility available fault current of 215000MVA and $Z_0 = 3.5*Z_1$ for system
First, build the sequence network

- $Z_{t1}=Z_{t2}=Z_{t0}=5\%$ on 50MVA base
- $Z_{s1}=1\text{pu}$ on 215000MVA base
- $Z_{s1}=1 \times (230/13)^2 \times 50/215000 = 0.0728\text{pu} @ 13\& 50$
- $I = V/Z = 1\text{pu} / (0.05\text{pu} + 0.05\text{pu} + 0.05\text{pu} + 0.0728 + 0.0728 + )$
- $I = 3.38\text{pu}$
- $I_{\text{base}} = 50\text{MVA} / (\sqrt{3} \times 13\text{KV}) = 2221\text{A}$
- $I = I_{\text{pu}} \times I_{\text{base}} = 7.51\text{KA}$
- $I_a = I_1 + I_2 + I_3 = 22\text{KA}$
What does this look like in the real world
Delta-Wye
Pre-Fault Values

F1 Symmetrical components

M1 Symmetrical components

Pre-fault all positive seq.
Fault Values

Why are the sequence values of M1 and F1 different???????
The Fault Network as Seen From F1
The Fault Network as Seen From M1

<table>
<thead>
<tr>
<th></th>
<th>Graph 2</th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>L1</td>
<td>51.52 A</td>
<td>-30.60°</td>
<td></td>
</tr>
<tr>
<td>L2</td>
<td>51.28 A</td>
<td>30.76°</td>
<td></td>
</tr>
<tr>
<td>L0</td>
<td>0.07 A</td>
<td>0.86°</td>
<td></td>
</tr>
</tbody>
</table>
What Effect Does this Fault Have on Voltage?
What Effect Does this Fault Have on Voltage?
Identification of Fault Types Quiz
Problem #1

Positive and Negative Sequence Current are equal, Zero Sequence is a different values, Positive, Negative and Zero sequence Voltage Equal.
Problem #2

Positive and Negative Sequence Current are equal, No Zero Sequence current
The V0 and V2 conundrum
POTT Scheme

POTT – Permissive Overreaching Transfer Trip

Communication Channel

End Zone
POTT Scheme

Local Relay
FWD \rightarrow GND
Local Relay-Z2

Remote Relay
FWD \rightarrow GND
Remote Relay-Z2

TRIP

Communication Channel

Local Relay
ZONE 2 PKP
OR
Ground Dir OC Fwd

Remote Relay
ZONE 2 PKP
OR
Ground Dir OC Fwd
Polarizing Quantities
Arc Impedance

\[ Z_L \]

No Load Fault Resistance

Load into Relay Bus

Load From Relay Bus
Solve the Arc Impedance Problem with Directional Overcurrent
Sequence V Generated by a fault
Faults Generate V0 and V1
Solution

In a weak in feed, not enough current for polarization
For remote faults not enough V0 for polarization.
Solution: use dual polarization.